

On the Gauss-Kronrod quadrature formula for a modified weight function of Chebyshev type

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In this paper, we consider the Gauss-Kronrod quadrature formulas for a modified Chebyshev weight. Efficient estimates of the error of these Gauss-Kronrod formulae for analytic functions are obtained, using techniques of contour integration that were introduced by Gautschi and Varga. Some illustrative numerical examples which show both the accuracy of the Gauss-Kronrod formulas and the sharpness of our estimations are displayed. Though for the sake of brevity we restrict ourselves to the first kind Chebyshev weight, a similar analysis may be carried out for the other three Chebyshev type weights; in the original paper, written in common by Miodrag Spalević, Ramon Orive, Ljubica Mihić and Aleksandar Pejčev, part of the corresponding computations are included in a final appendix.

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