Some new results concerning the classical Bernstein cubature formula

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We present a solution to the approximation problem of the volume obtained by the integration of a bivariate function when a double integral cannot be computed exactly. The approximation of various double integrals can be done by a few cubature formulas (for instance, the Newton-Cotés cubature formulas) according to the specialty literature. Constructed by means of the bivariate Lagrange polynomial, trapezoidal and Simpson cubature formulas use a fixed number of nodes, resulting in a single possible approximation for a double integral. In order to be more flexible with this fact, we bring to the light a cubature formula constructed on the base of the classical bivariate Bernstein operator. As a valuable tool to approximate any volume resulted by integration of a bivariate function, we use the classical Bernstein cubature formula

$$\int_a^b \int_c^d F(x,y)dxdy \approx \frac{(b-a)(d-c)}{(n_1+1)(n_2+1)} \sum_{k_1=0}^{n_1} \sum_{k_2=0}^{n_2} F\left(a + \frac{k_1(b-a)}{n_1}, c + \frac{k_2(d-c)}{n_2}\right),$$

obtained as a continuation of our sustained research in [1], [2] and [3]. If the bivariate interval $[a,b] \times [c,d]$ (the bivariate symmetrical interval $[-a,a] \times [-a,a]$) is large, then the classical composite Bernstein cubature formula is suitable for the approximation of a double integral. Numerical examples are given to increase the validity of the theoretical aspects.

References

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