Error Estimates for Certain Quadrature Formulae

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Here we studied the error bound of Gauss-Legendre quadrature for analytic functions. The basic idea is to express the remainder of Gauss-Legendre quadrature as a contour integral, then the error bound is reduced to find the maximum of the kernel function:

$$K_n(z;\omega) = \frac{\varrho_n(z;\omega)}{\pi_n(z)}, \quad \varrho_n(z;\omega) = \int_{-1}^{1} \frac{\pi_n(t)}{z-t}\,dt, \quad z \in \mathbb{C} \setminus [-1,1]. \tag{1}$$

Inspired by the work of [1] and applying the results of [2], we obtained explicit and asymptotic formula of the kernel function $K_n(z;\omega)$ as $\rho \to \infty$. Explicit expression is used for determining location on the ellipses where maximum of the modulus of the kernel is attained.

Keywords: Gauss quadrature formulae, Legendre polynomials, remainder term for analytic function, error bound

References

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