An iterative approach for a trigonometric Hermite interpolant

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In [2] it was introduced a trigonometric barycentric interpolant of an arbitrary 2π-periodic function in $[0, 2\pi)$ on some ordered nodes $0 \leq \theta_0 < ... < \theta_{n-1} < 2\pi$ which converges exponentially when the nodes are equidistant points or their images under a periodic conformal map [1] and has a logarithmic growth of the Lebesgue constant for a wide class of nodes [3]. We present here an iterative method to construct a trigonometric Hermite interpolant based on the latter interpolant. In fact, by using the auxiliary function

$$d_i(\theta) = 2\sin\left(\frac{x - x_i}{2}\right)$$

and the basis function $b_i(\theta)$ of the interpolant, it is possible to construct in an iterative way, similarly as done in [4] for the Floater-Hormann family of interpolant, the Hermite interpolant by considering

$$b_{i,j}(\theta) = \frac{1}{j!} d_i(\theta)^j b_i(\theta)^{j+1}$$

and therefore the interpolant

$$r_j(\theta) = \sum_{i=0}^{n} \sum_{j=0}^{m} b_{i,j}(x) g_{i,j}$$

where

$$g_{i,0} = f(\theta_i) \qquad g_{i,j} = f^{(j)}(\theta_i) - r_{j-1}^{(j)}(\theta_i).$$

Furthermore, to implement it numerically we compute the differential matrix of the resulting interpolant at each iteration.
Finally, we are going to present some numerical tests.

References

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