Gaussian rule for integrals involving Bessel functions

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In this work we present a Gaussian type quadrature rule for the evaluation of integrals involving fractional powers, exponentials and Bessel functions of the first kind. In general, the technique commonly used in the computation of the coefficients of the three-term recurrence relation, for the corresponding orthogonal polynomials, is the Chebyshev algorithm (see [2, sect.2.3]). Nevertheless, it is well known (see e.g. [4]) that the computation of the recurrence coefficients can be inaccurate for growing number of quadrature points because the problem is severely ill conditioned. This issue can be partially overcame by using the modified moments (see [5], [2, sect.2.4], [6]), having at disposal a family of polynomials orthogonal with respect to a weight function similar to the one of the problem. This approach can be efficient in general but not always when working with unbounded intervals of integration (see [3] and [4]). In this framework, we present an alternative approach that is based on the preconditioning of the moment matrix. In particular, since the three-term recurrence coefficients can be written in terms of ratios of determinants of the moments matrix or slight modification of them (see [1, sect.2.7]), we exploit the Cramer rule to show that the coefficients can be computed by solving a linear system with the moment matrix. Since the weight function of the problem can be interpreted as a perturbation of the weight function of the generalized Laguerre polynomials, we use the moment matrix of these polynomials as preconditioner. The numerical experiments confirm the reliability of this approach and shows that it is definitely more stable than the modified Chebyshev algorithm.

References

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