Types of convergence which preserve continuity

Simon Reinwand

Department of mathematics, University of Würzburg (Germany)
e-mail sranthrop@gmx.net

It is well known that a locally uniformly convergent sequence of continuous functions always has a continuous limit function, whereas a pointwise convergent sequence of continuous functions may have a discontinuous limit.
This raises the question what additional assumption on top of pointwise convergence can be made in order to guarantee that the limit function of a sequence of continuous functions is always continuous. The first one who solved this problem was Arzelá in 1883, who introduced the notion of *quasi uniform convergence*.
In this talk we investigate - in addition to pointwise and locally uniform convergence - three further types of convergence in metric spaces, namely *quasi uniform*, *semi uniform* and *continuously uniform* convergence. We give criteria under which a sequence converges in one of these types and keep our eyes on those which preserve continuity.
In addition, we point out that several types of convergence can be used to characterize compactness.
As an application we use some of the theoretical results to the discussion of (autonomous) composition operators in the space BV of real-valued functions of bounded variation. There are known criteria guaranteeing that these operators map BV into itself. However, pointwise continuity of these operators is a delicate problem with an interesting history.
We present criteria under which sequences of composition operators converge locally uniformly and semi uniformly in the space BV. Moreover, we sketch a new and short proof of the pointwise continuity of such operators using semi uniform convergence.
Apart from recalling known and discussing new results we put a particular emphasis on examples and counter examples.

Sampling Kantorovich algorithm for the detection of Alzheimer's disease

Danilo Costarelli [a], Marco Seracini [b], **Arianna Travaglini**[a,c], Gianluca Vinti[a]

[a] Department of Mathematics and Computer Science, University of Perugia (Italy)
[b] Department of Physics and Astronomy "Augusto Righi", University of Bologna (Italy)
[c] Department of Mathematics and Computer Science "U. Dini", University of Florence (Italy)
danilo.costarelli@unipg.it, marco.seracini2@unibo.it, arianna.travaglini@unifi.it, gianluca.vinti@unipg.it

Among sampling-type operators, the Sampling Kantorovich operator represents a useful tool for dealing with discontinuous functions [2]. Its muldimensional version has been implemented and allows not only to reconstruct, but also to enhance the resolution of images, as it acts boths as a low-pass filter and as a magnifier, increasing spatial resolution of images [4]. Indeed, Sampling Kantorovich algorithm has been used, with satisfactory results, to both biomedical and engineering fields [1, 3].

The talk is focused on some recent results, which consist in the use of different algorithms, including Sampling Kantorovich algorithm, to process magnetic resonance images for the identification of biomarkers for Alzheimer's disease. The quality of reconstruction is evalueted, comparing the volumetric values of the images processed with the various algorithms, with the ground truth values, considered as reference. Moreover, the stereological Cavalieri\Point counting technique is used to infer volumetric data, starting from the knowledge of the planar sections.

References

[1] F. Asdrubali, G. Baldinelli, F. Bianchi, D. Costarelli, A. Rotili, M. Seracini, G. Vinti, Detection of thermal bridges from thermographic images by means of image processing approximation algo rithms, Applied Mathematics and Computation 317 (2018), pp. 160-171.

[2] C. Bardaro, P.L. Butzer, R.L. Stens, G. Vinti, Kantorovich-Type Generalized Sampling Series in the Setting of Orlicz Spaces, Sampling Theory in Signal and Image Processing, 9 (6) (2007), 29-52.

[3] C. D. Costarelli, M. Seracini, G. Vinti, A segmentation procedure of the pervious area of the aorta artery from CT images without contrast medium, Mathematical Methods in the Applied Sciences 43 (1), (2020), 114-133.

[4] D. Costarelli, G. Vinti, Approximation by Multivariate Generalized Sampling Kantorovich Operators in the Setting of Orlicz Spaces, Bollettino U.M.I., 9 (4) (2011).