Quantitative estimates for nonlinear sampling Kantorovich operators in functional spaces

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In real world applications, signals can be suitably reconstructed by nonlinear procedures; this justifies the study of nonlinear approximation operators. A wide literature can be found in [5, 1, 6, 4, 2, 3].

Herein, we present some quantitative estimates for the nonlinear sampling Kantorovich operators in the multivariate setting using the modulus of smoothness of $L^\varphi(\mathbb{R}^n)$. As a consequence, the qualitative order of convergence can be obtained, in case of functions belonging to suitable Lipschitz classes. The general frame of Orlicz spaces allows us to deduce the corresponding estimates in several instances of well-known and useful spaces, as L^p-spaces, Zygmund spaces and exponential spaces. Moreover, in the particular case of L^p-spaces, we also obtain a direct estimate that is sharper than that one achieved in the general case of Orlicz spaces, thanks to the properties of the modulus of smoothness in L^p. Several examples of nonlinear multivariate sampling Kantorovich operators, by using some special kernels, are provided.

References

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