Nonlinear composition operators in Grand Lebesgue spaces

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Let Ω be an open subset of $\mathbb{R}^n$ of finite measure. Let f be a Borel measurable function from $\mathbb{R}$ to $\mathbb{R}$. We prove necessary and sufficient conditions on f in order that the composite function $T_f[g] = f \circ g$ belongs to the Grand Lebesgue space $L_{p),\theta}(\Omega)$ whenever g belongs to $L_{p),\theta}(\Omega)$.
We also study continuity, uniform continuity, Hölder and Lipschitz continuity of the composition operator T_f in $L_{p),\theta}(\Omega)$.

Accurate computation of the multidimensional fractional Laplacian

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In this talk we discuss approximation formulas for the fractional Laplacian $(-\Delta)^{\alpha/2}$, $0 < \alpha < 2$, in the framework of the method approximate approximations. The fractional Laplacian appears in different fields of mathematics (PDE, harmonic analysis, semi- group theory, probabilistic theory) as well as in many applications (optimization, finance, materials science, water waves). If we introduce the convolution

$$\mathcal{N}_\alpha(f)(\mathbf{x}) = c_{n,\alpha} \int_{\mathbb{R}^n} \frac{f(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|^{n-2+\alpha}} d\mathbf{y}, \quad c_{n,\alpha} = \frac{2^{\alpha-2}}{\pi^{n/2}} \frac{\Gamma(\frac{n-2+\alpha}{2})}{\Gamma(\frac{2-\alpha}{2})}, \tag{1}$$

then the fractional Laplacian can be represented as the ordinary Laplacian of the volume potential $\mathcal{N}_\alpha f$,

$$(-\Delta)^{\alpha/2} f(\mathbf{x}) = -\Delta \mathcal{N}_\alpha(f)(\mathbf{x}) \,. \tag{2}$$

We propose a method of an arbitrary high order for the approximation of $\mathcal{N}_\alpha f$ and $(-\Delta)^{\alpha/2} f$, $n \geq 3$, which is based on the approximation of the function f via the basis functions introduced by approximate approximations (cf. [2]), which are product of Gaussians and special polynomials. Then the n-dimensional integral (1) applied to the basis functions is represented by means of a one-dimensional integral where the integrand has a separated representation, i.e., it is a product of functions depending only on one of the variables.
This construction enables to obtain one-dimensional integral representations with separated integrand also for the fractional Laplacian (2), when applied to the basis functions. An

accurate quadrature rule and a separated representation of the density f provide a separated representation for $\mathcal{N}_\alpha f$ and $(-\Delta)^{\alpha/2} f$. Thus, only one-dimensional operations are used and the resulting approximation procedure is fast and effective also in high-dimensional cases, and provides approximations of high order, up to a small saturation error. We prove error estimates and report on numerical results illustrating that our formulas are accurate and provide the predicted convergence rate $2, 4, 6, 8$ (cf. [1]).

References

[1] F. Lanzara, V. Maz'ya, G. Schmidt, *Fast computation of the multidimensional fractional Laplacian*, Applicable Analysis, 2021. (https://doi.org/10.1080/00036811.2021.1986025)

[2] V. Maz'ya, G. Schmidt, *Approximate approximations*, AMS, Providence (RI), 2007.