Riesz-Fisher maps, Semiframes and Frames in rigged Hilbert spaces

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Given a Hilbert space $\mathcal{H}$, a sequence of vectors $\{f_n\}$ in $\mathcal{H}$ is a frame if there exists $A, B > 0$ such that :

$$A\|f\|^2 \leq \sum_{k=1}^{\infty} |< f \mid f_n >|^2 \leq B\|f\|^2, \quad \forall f \in \mathcal{H}.$$

As known, frames are generalizations of orthonormal bases, and their versatility is the motivation of the crescent importance in applications (signal analysis, image processing...) and in various areas of pure mathematics (time-frequence analisis, sampling theory, ...). However, this framework in Hilbert space does not include the case of *generalized eigenvectors* $\{\omega_x\}_{x\in X}$ (i.e. eigenvectors of an essetially self-adjoint operator A on $\mathcal{D} \subset \mathcal{H}$, x varies in some measure space X) that does not belongs to $\mathcal{H}$, but that can be viewed as distributions.That is the case of eigenvectors of continuous spectrum in QM.This motivates the extension of frames and bases to a rigged Hilbert space, that is the triplet:

$$\mathcal{D} \subset \mathcal{H} \subset \mathcal{D}^{\times}$$

where $\mathcal{D}$ is a locally convex space continuously embedded in $\mathcal{H}$ and $\mathcal{D}^{\times}$ the conjugate dual of $\mathcal{D}$.

References

[1] Trapani, C.; Triolo S., Tschinke F., *Distribution Frames and Bases*, J. Fourier Anal. and Appl. 25, pp. 2109-2140 (2019);

[2] Corso, R., Tschinke, F., Some notes about distribution frame multipliers, in Landscapes of Time-Frequency Analysis-ATFA 2019, Vol.2, 103-122, Appl. Numer. Harmon. Anal., Boggiatto P., Bruno T., Cordero E., Feichtinger, H.G., Nicola F., Birkhüser, Cham, 2020.

[3] Tschinke, F., Riesz-Fischer maps, Semiframes and frames in rigged Hilbert spaces, in *Operator theory, Functional Analysis and Applications*, 625-645, Opertor theory: Advances and Applications 282; Bastos, M.A., Castro, L.Karlovich, A.Y., Eds.;Birkhäuser: Cham, Switzerland, 2021.