Regularity of global solutions of PDE via time-frequency methods

Claudio Mele [a], **Alessandro Oliaro**[b]

[a] Department of Mathematics and Physics "E. De Giorgi", University of Salento (Italy)
[b] Department of Mathematics "G. Peano", University of Torino (Italy)
claudio.mele1@unisalento.it, alessandro.oliaro@unito.it

We present some results, contained in [3], on regularity of linear partial differential operators with polynomial coefficients in non isotropic ultradifferentiable classes. The problem of regularity was first introduced by Shubin in the frame of Schwartz functions and tempered distributions; a linear operator $A : \mathcal{S}' \to \mathcal{S}'$ is said to be regular if the conditions $u \in \mathcal{S}'$, $Au \in \mathcal{S}$ imply that $u \in \mathcal{S}$. Shubin formulates an hypoellipticity condition (in his global pseudodifferential calculus), that is sufficient to have regularity of the correponding operator. On the other hand, such hypoellipticity is far to be necessary, as there are several examples of operators which are not hypoelliptic but are regular (such as the Twisted Laplacian). The problem of characterizing regularity for classes of operators is quite hard. Even in very particular cases (as for ordinary differential operators with polynomial coeffcients) necessary and sufficient conditions for regularity are not known. Various results have been obtained in this field, showing classes of Partial Differential Operators that are regular, both in the classical sense and in scales of ultradifferentiable spaces, cf. [1, 2, 4, 5]. In this work we study regularity of partial differential equations with polynomial coefficients in non isotropic Beurling spaces of ultradifferentiable functions of global type. We study the action of transformations of Gabor and Wigner type in such spaces and we prove that a suitable representation of Wigner type allows to prove regularity for classes of operators that do not have classical hypoellipticity properties.

References

[1] C. Boiti, D. Jornet, A. Oliaro, *Regularity of partial differential operators in ultradifferentiable spaces and Wigner type transforms*, J. Math. Anal. Appl. 446 (2017), 920-944.

[2] E. Buzano, A. Oliaro, *Global regularity of second order twisted differential operators*, J. Differential Equations 268, 7364–7416 (2020).

[3] C. Mele, A. Oliaro, *Regularity of global solutions of partial differential equations in non isotropic ultradifferentiable spaces via time-frequency methods*, J. Differential Equations 286, 821–855 (2021).

[4] F. Nicola, L. Rodino, *Global regularity for ordinary differential operators with polynomial coefficients*, J. Differential Equations **255** (2013), no. 9, 2871–2890.

[5] M.-W. Wong, *Weyl Transforms, the Heat Kernel and Green Functions of a Degenerate Elliptic Operator*, Ann. Global Anal. Geom., **28**, 3 (2005), 271–283.