Existence Theory For Some Class Of Nonlocal Integro-Differential Inclusions without Compactness or Norm-Continuity

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Abstract A Gabor system $\mathcal{G}(g, a, b)$ is a sequence of the type $\{g_{hk} = e^{2\pi i\, bk\cdot x} g(x-ah)\}_{h,k\in\mathbb{Z}^d}$, with g measurable function on $\mathbb{R}^d$, $a, b > 0$.
$\mathcal{G}(g, a, b)$ is said to be a frame in $L^2(\mathbb{R}^d)$ if $A\|f\|^2_{L^2} \leq \sum_{h,k\in\mathbb{Z}^d} |(f, g_{h,k})|^2 \leq B\|f\|^2_{L^2}$, for some $A, B > 0$ and any $f \in L^2(\mathbb{R}^d)$. Gabor frames play an important role in signal processes.
A wide literature is devoted in finding conditions on the window g and the lattice parameters $a, b > 0$, which allow the corresponding Gabor system to be a frame in $L^2(\mathbb{R}^d)$ so that the Gabor operator $S_{g,g}f = \sum_{h,k\in\mathbb{Z}^d}(f, g_{hk})g_{hk}$ is invertible in $\mathcal{L}(L^2)$ and a reconstruction formula $f = \sum_{h,k\in\mathbb{Z}^d}(f, g_{hk})\gamma_{h,h}$ is available, with $\gamma = S^{-1}_{g,g}g$. To this respect the very basic assumption is that a and b are "small enough".
In this talk we introduce results of continuity and invertibility in $L^p(\mathbb{R}^d)$ for pseudodifferential operators with symbols $\sigma(x, \xi)$ periodic in both the variable, which allow us to obtain sufficient conditions for the invertibility of $S_{g,g}$.

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