Do properties of frame-related sequences spread in a scale of Hilbert spaces?

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Frames have been introduced and studied as a powerful alternative to Hilbert space bases and they allow a deep theory. Also, they are very important for applications e.g. in signal analysis and in physics. There exist many notions of frame-related sequences, generalizing the notion of basis in Hilbert space. Both in mathematics and physics it is natural to consider a full scale of spaces, and not only a single one. Then a question arises: do properties of frame-related sequences in a space of a scale of Hilbert spaces, such as either completeness or the property of being a (semi-)frame, spread in a scale of Hilbert spaces? It has been found [1] that the answer is not always affirmative. Sometimes it is either affirmative, as for completeness, or partially affirmative, as for the property of being a lower (upper) semi-frame which is kept in larger (smaller) spaces of the scale, or negative, as the property of being a frame: a sequence cannot be a frame for both two different Hilbert spaces of a certain scale of Hilbert spaces.

Joint work with: Peter Balazs and Hessam Hosseinnezhad

1. P. Balazs, G. Bellomonte, H. Hosseinnezhad, Frame-Related Sequences in Chains and Scales of Hilbert Spaces, Axioms, 11 (4) (2022) art. n. 180 (20 pp.)