Representation of Operators Using Fusion Frames

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To solve operator equations numerically, matrix representations are used employing bases [4] or more recently frames[2]. For finding the numerical solution of operator equations [5] a decomposition in subspaces is needed in many applications. To combine those two approaches, it is necessary to extend the known method of matrix representation to the utilization of fusion frames [1].

We investigate this representation of operators on a separable Hilbert space with Bessel fusion sequences, fusion frames and Riesz decompositions [3]. Let $\{W_i\}_{i\in I}$ be a family of closed subspaces of $\mathcal{H}$ and $\{w_i\}_{i\in I}$ be a family of weights, i.e. $w_i > 0, i \in I$. The sequence $W = (W_i, w_i)$ is called a fusion frame for $\mathcal{H}$ if there exist constants $0 < A_W \leq B_W < \infty$ such that

$$A_W\|f\|^2 \leq \sum_{i\in I} w_i^2 \|\pi_{W_i} f\|^2 \leq B_W\|f\|^2, \qquad (f \in \mathcal{H}).$$

For two fusion frames W and V we define a matrix representation not only in a canonical but also in an alternate way - taking the particular property of the duality of fusion frames into account.

We will give the basic definitions and show some structural results, like that the functions assigning the alternate representation to an operator is an algebra homomorphism. We give formulas for pseudo-inverses and the inverses (if existing) of such matrix representations. We apply this idea to Schatten-class operators. Consequently, we show that tensor products of fusion frames are frames in the space of Hilbert-Schmidt operators.

We show how this can be used for the solution of operator equations and link our approach to the additive Schwarz algorithm. We provide small proof-of-concept numerical experiments. Finally we show the application of this concept to overlapped convolution and the non-standard wavelet representation.

References

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