Construction of a sequence of orthogonal rational functions

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Orthogonal polynomials are an important tool to approximate functions. Orthogonal rational functions provide a powerful alternative if the function of interest is not well approximated by polynomials.
Polynomials orthogonal with respect to certain discrete inner products can be constructed by applying the Lanczos or Arnoldi iteration to appropriately chosen diagonal matrix and vector. This can be viewed as a matrix version of the Stieltjes procedure. The generated nested orthonormal basis can be interpreted as a sequence of orthogonal polynomials. The corresponding Hessenberg matrix, containing the recurrence coefficients, also represents the sequence of orthogonal polynomials.
Alternatively, this Hessenberg matrix can be generated by an updating procedure. The goal of this procedure is to enforce Hessenberg structure onto a matrix which shares its eigenvalues with the given diagonal matrix and the first entries of its eigenvectors must correspond to the elements of the given vector. Plane rotations are used to introduce the elements of the given vector one by one and to enforce Hessenberg structure.
The updating procedure is stable thanks to the use of unitary similarity transformations. In this talk rational generalizations of the Lanczos and Arnoldi iterations are discussed. These iterations generate nested orthonormal bases which can be interpreted as a sequence of orthogonal rational functions with prescribed poles. A matrix pencil of Hessenberg structure underlies these iterations. We show that this Hessenberg pencil can also be used to represent the orthogonal rational function sequence and we propose an updating procedure for this case. The proposed procedure applies unitary similarity transformations and its numerical stability is illustrated.