A New Method for the Construction of Schur Stable Matrix Families

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This study is concerned with the construction of some Schur stable matrices families consisting of linear and convex combinations of matrices $A \in S_N$ and $B \in M_N(\mathbb{C})$, where $S_N = \{A \in M_N(\mathbb{C}) \mid |\lambda_i(A)| < 1\}$ and $M_N(\mathbb{C}) = \{A \mid N \times N,\ a_{ij} \in \mathbb{C}\}$. It is well known that, according to Lyapunov's theorem, a necessary and sufficient condition for the matrix A to be Schur stable is that the Lyapunov matrix equation $A^*HA - H + I = 0$ has a Hermitian and positive definite solution H [4]. On the other hand, the parameter ω which indicates the Schur stability and the quality of the matrices, is defined as $\omega(A) = \|H\| \geq 1$. If $\omega(A) < \infty$ then A is Schur stable, otherwise it is not [1, 3, 4]. Given a specified parameter $\omega^*\,(> 1)$, if $\omega(A) \leq \omega^*$ then the matrix A is ω^*-Schur stable. In this talk, matrix families $\mathcal{A}_1$ and $\mathcal{A}_2$ will be introduced by linear and convex combinations of matrices $A \in S_N$ and $B \in M_N(\mathbb{C})$, respectively. In addition, some theorems and results about the Schur stability of these families will be given. A new method based on the Schur stability parameter and the continuity theorems, which indicates the sensitivity of the Schur stability, will also be described [2, 5]. According to this method, intervals $\mathcal{R}_1$ and $\mathcal{R}_2$ are obtained, which guarantee the Schur stability and ω^*-Schur stability of the matrix families $\mathcal{A}_1$ and $\mathcal{A}_2$, respectively. Finally, illustrative examples related to the subject will be given.

References

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