Perron communicability and sensitivity of multilayer networks

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Modeling complex systems that consist of different types of objects leads to multilayer networks, where nodes in the different layers represent different kind of objects. Nodes are connected by edges, which have positive weights. De Domenico et al. [2] describe how multilayer networks with a set of N nodes and L layers can be represented by a supra-adjacency matrix $B \in \mathbb{R}^{NL\times NL}$. It is the purpose of this talk to carry out an investigation that focuses on the sensitivity of the network communicability (see, e.g., [1], [3]) to perturbations in the multilayer network, by studying the sensitivity of the Perron root of B.
In case of layer-coupled multiplex networks, in which nodes in different layers are identified with each other, one has

$$B = \mathrm{diag}[A^{(1)}, A^{(2)}, \ldots, A^{(L)}] + \mathbf{1}_L\mathbf{1}_L^T \otimes I_N - I_{NL},$$

where $A^{(\ell)} \in \mathbb{R}^{N\times N}$, with $\ell = 1, 2, \ldots, L$, is the non-negative adjacency matrix associated with the graph for the ℓth layer. Here, $\mathbf{1}_L \in \mathbb{R}^L$ denotes the vector of all entries one and $\otimes$ the Kronecker product. Such particular structure of the supra-adjacency matrices associated with multiplexes is exploited in the relevant structured eigenvalue sensitivity analysis; see [4]. Finally, as in [5], the network analysis we carry out sheds light on which edge weights to make larger to increase the communicability of the network, and which edge weights can be made smaller or set to zero without affecting the communicability significantly.

References

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