A projection method for eigenvalue problems of linear nonsquare matrix pencils

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Consider the computation of all the finite eigenvalues of a linear matrix pencil $zB - A \in \mathbb{C}^{m\times n}$, $z \in \mathbb{C}$, A, $B \in \mathbb{C}^{m\times n}$ in a given simply connected open set $\Omega \subset \mathbb{C}$

$$A\boldsymbol{x} = \lambda B\boldsymbol{x}, \quad \boldsymbol{x} \in \mathbb{C}^n \setminus \{\boldsymbol{0}\}, \quad \lambda \in \Omega \tag{1}$$

and the corresponding eigenvectors. In a class of eigensolvers, such as the Sakurai–Sugiura method [4] and the FEAST algorithm [3], a complex moment consisting of a resolvent filters out undesired eigencomponents and extracts the desired ones in a pseudo-random matrix. Thus, methods of this kind project a regular matrix pencil onto the eigenspace associated with eigenvalues in a prescribed region and give the eigenvalues and the corresponding eigenvectors of a regular matrix pencil. This study extends a projection method for regular eigenproblems [5, 1] to the singular nonsquare case [2]. The extended method involves complex moments given by the contour integrals of generalized resolvents associated with nonsquare matrices. We establish conditions such that the method gives all finite eigenvalues in a prescribed region in the complex plane. In numerical computations, the contour integral is approximated by a numerical quadrature, similarly to the regular case. Each quadrature point gives a least squares problem to solve, and it can be solved independently. The primary cost lies in the solutions of linear least squares problems that arise from quadrature points, and they can be readily parallelized in practice. Numerical experiments on large matrix pencils illustrate this method. The new method is more robust and efficient than previous methods, and based on experimental results, it is conjectured to be more efficient in parallelized settings.

References

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