Orthogonal Cauchy-like matrices

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A matrix $C \in \mathbb{R}^{n\times n}$ is Cauchy if its entries C_{ij} have the form

$$C_{ij} = \frac{1}{x_i - y_j}, \qquad i, j = 1, \ldots, n,$$

where x_i, y_j for $i, j = 1, \ldots, n$ are mutually distinct real numbers. Besides to their pervasive occurrence in computations with rational functions, Cauchy matrices play an important role in deriving algebraic and computational properties of many relevant structured matrix classes [1]. Indeed, they occur as fundamental blocks (together with trigonometric transforms) in decomposition formulas and fast solvers for Toeplitz, Hankel, and related matrices, see e.g., [2].

The main result of this contribution is to provide a complete description of the set of orthogonal Cauchy-like matrices, that is, the orthogonal matrices $K \in \mathbb{R}^{n\times n}$ with entries

$$K_{ij} = \frac{a_i b_j}{x_i - y_j}, \qquad i, j = 1, \ldots, n.$$

Interest in these matrices arises from the paper [3], where orthogonal matrices obtained by scaling rows and columns of Cauchy matrices are needed in the design of allpass filters for signal processing purposes. We illustrate their relationships with secular equations, the diagonalisation of symmetric quasiseparable matrices and the construction of orthogonal rational functions with free poles. Moreover, we characterize matrix families that are simultaneously diagonalized by orthogonal Cauchy-like matrices.

References

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[3] S. J. Schlecht, *Allpass feedback delay networks*, IEEE Trans. Signal Process. 69 (2021), pp. 1028–1038.