Efficient Approximation of Singular BVPs in ODEs

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We deal with boundary value problems for systems of ordinary differential equations with singularities. Typically, such problems have the form

$$z'(t) = F(t, z(t)), \quad t \in (0, 1], \quad B_0 z(0) + B_1 z(1) = \beta,$$

where $\lim_{t\to 0} F(t, z(t)) = \infty$ and $\lim_{t\to 0} \partial F(t, z)/\partial z = \infty$. The analysis is usually done for the model equation

$$z'(t) = \frac{1}{t^\alpha} M z(t) + f(t, z(t)), \quad t \in (0, 1], \quad B_0 z(0) + B_1 z(1) = \beta,$$

where $f(t, z)$ may also be in the form of $g(t, z)/t$ with a smooth function $g(t, z)$. For $\alpha = 1$ the problem has a *singularity of the first kind*, while for $\alpha > 1$ the singularity is commonly referred to as *essential singularity.* We briefly recapitulate the analytical properties of the above problems with a special focus on the most general boundary conditions which guarantee their well-posedness.

To compute the numerical approximation for z we use polynomial collocation, because the method retains its high convergence order even in case of singularities. The usual high-order superconvergence at the mesh points does not hold in general. However, the uniform superconvergence is preserved (up to logarithmic factors). We will discuss how the collocation performs for problems with the inhomogeneity of the form $g(t, z)/t$.

The updated version of the MATLAB code `bvpsuite1.1` with the special focus on the above problem class has been implemented. For higher efficiency, estimate of the global error and adaptive mesh selection are provided. The code can be applied to arbitrary order problems in implicit form. Also systems of index 1 differential-algebraic equations (DAEs) are in the scope of the code. We illustrate the performance of the software with a special focus on parameter-dependent problems by means of numerical simulation of models in applications.

References

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