Pseudospectral approximation of characteristic roots of equations with infinite delay

Francesca Scarabel [a], **Rossana Vermiglio**[b]

[a] Department of Mathematics, The University of Manchester, Oxford Rd, M13 9PL, Manchester, UK

[b] CDLab - Computational Dynamics Laboratory, Department of Mathematics, Computer Science and Physics, University of Udine, Via delle Scienze 206, 33100, Italy,

e-mail: francesca.scarabel@manchester.ac.uk, rossana.vermiglio@uniud.it

Delay equations with infinite delay (iDEs) are widely used in mathematical biology, and, in this context, the interest is often focused on the stability of equilibria as well as on their bifurcation analysis. But iDEs generate infinite-dimensional dynamical systems and so some numerical methods are needed. The pseudospectral discretization (PSD) has been successfully applied for the numerical stability analysis of equilibria, and for the numerical bifurcation in various contexts. Here we consider delay differential and renewal equations with infinite delay, for which the principle of linearized stability ensures that the stability of an equilibrium can be inferred from the eigenvalues of the infinitesimal generator of the linearization at the equilibrium [1]. By introducing a general abstract framework that encompasses both types of equations, we consider the PSD approach based on exponentially weighed polynomial interpolation at near-optimal Laguerre zeros. Some numerical tests illustrate the convergence of the characteristic roots for linear iDEs and the effectiveness of the technique for the bifurcation analysis of nonlinear equations [2, 3].

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References

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