Matrix-Oriented Discretization Methods for Evolutionary Problems

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An interesting class of evolutionary problems is given by reaction-diffusion PDE systems where the coupling between diffusion and nonlinear kinetics can lead to the so-called Turing instability. In this case, a variety of spatial patterns can be attained as stationary solutions for longtime integration. To capture the morphological peculiarities of the Turing patterns, a very fine space discretization may be required, limiting the use of standard (vector-based) ODE solvers in time because of excessive computational costs.
We show that the structure of the diffusion matrix can be exploited to build matrix-oriented (MO) versions of some classical time integrators. In particular, we consider finite differences on square domains and classical Lagrangian FEM on x-normal domains and even on special surfaces. In the first case, the discrete problem is then reformulated as a sequence of Sylvester matrix equations, that we solve by the *reduced approach* in the associated spectral space [?, ?]. On general domains, at each time step, *multiterm Sylvester matrix equations* must be solved, where the additional terms account for the geometric contribution of the domain shape. In this case, we solve the matrix equations by the matrix-oriented form of the Preconditioned Conjugate Gradient (MO-PCG) method [?].
We illustrate our findings for the reaction-diffusion DIB model describing metal growth during battery charging processes. We apply the MO-IMEX Euler scheme for the approximation of stationary Turing patterns on rectangular domains and cylindrical surfaces. We will show encouraging results in terms of execution times and memory storage. For this reason, we present some initial recent results also for the efficient approximation of oscillatory solutions, like spiral waves and Turing-Hopf patterns, by some MO-splitting schemes.

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