Numerical solution of the fractional diffusion equation by spline approximations

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In recent years, fractional differential equations have occupied a prominent place in modeling anomalous diffusion, i.e., transport processes where long-range correlations between particles or anomalous long jumps of particles can occur. Anomalous diffusion has been observed in a variety of materials, such as porous media, biological tissues, condensed matter [?]. At the same time, the demand of efficient numerical methods for solving diffusion differential problems with fractional derivatives has increased enormously.
In this talk, we present a numerical method suitable to solve fractional diffusion problems. For the time derivative we consider the Caputo derivative since it retains many peculiar features of the classical derivative [?]. As for the space derivative, we use the symmetric Riesz-Caputo derivative since it is more suitable to model transport processes in which contributions from both sides of the spatial domain have to be taken into account.
To solve the differential problem, we approximate its solution by a spline expansion [?], whose coefficients are evaluated by a collocation method [?, ?]. Spline approximations of operator equations can be efficiently evaluated exploiting the explicit expression of both classical and fractional derivatives of the spline basis [?, ?, ?]. We present some numerical tests to show the performance of the proposed method.

This is a joint work with E. Pellegrino and C. Sorgentone.

References

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