An extension of nonstandard finite differences with application to a vegetation model

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Discretization schemes based on NonStandard Finite Differences (NSFD) are a modification of Standard Finite Differences (SFD) schemes in which the classical denominators Δt and Δx (and also the related powers, if present) are replaced by particular scalar *denominator functions* satisfying certain conditions. Furthermore, some terms of the SFD schemes can be approximated in the NSFD methods with *non-local representations* (see, e.g., [2] and [7]). The goal of these techniques is to improve the stability of SFD schemes built for the solution of ordinary and partial differential equations, being also able to preserve the positivity and the equilibrium points properties of the continuous model.
In this talk [3], we extend the classical NSFD methodology by allowing the use of non-scalar denominator functions, inspired by Time-Accurate and highly-Stable Explicit operators (see [1]), also showing the connections between NSFD and exponentially fitted numerical methods (see, e.g, [4] and [6]), thanks to which it is possible to preserve the oscillation frequency, if a-priori known, of the exact continuous model solution. Finally, we apply the generalized NSFD methodology to a vegetation linear-diffusion non-linear-reaction model [5], showing through numerical tests the advantages of the proposed numerical technique.

References

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