Investigating the stability of periodic neutral renewal equations via Floquet multipliers

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The main classes of retarded functional equations are delay differential equations, which describe the solution by prescribing the values of its derivative, and renewal equations, which directly prescribe the values of the solution itself. Renewal equations proper are typically formulated as integral equations; neutral renewal equations (NRE), instead, involve also values of the solution at discrete times in the past. Even though NRE are quite important in biological modeling, until the recent work [1] little was known about their dynamics.

In this work we investigate the stability of autonomous and periodic NRE of the type $x(t) = f(t)x(t - \tau)$ from a theoretical point of view, performing numerical experiments both to exemplify our findings and to guide our investigation.

Our numerical approach is based on the pseudospectral collocation technique of [2, 3] for discretizing monodromy operators and computing Floquet multipliers, although a proof of Floquet theory for NRE is currently lacking.

References

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