Self Starting General Linear Methods for Ordinary Differential Equations

Giuseppe Izzo[a], Zdzislaw Jackiewicz[b]

[a] Dep. of Mathematics and Applications "R. Caccioppoli", University of Napoli Federico II (Italy)
[b] Department of Mathematics, Arizona State University (USA)
giuseppe.izzo@unina.it, jackiewicz@asu.edu

We have recently focused our attention on using general linear methods (GLMs) as a framework to analyze and generalize existing classes of numerical methods for ordinary differential equations. In this work we present the class of Self Starting GLMs, whose name point out one of their main features. Indeed, although they are multi-stage multi-step methods, they do not require any additional starting procedure. In particular, after presenting the general formulation, we focus on a subclass with a structure that is very similar to Runge-Kutta methods. With this approach, we show how some properties of these last methods can be improved, keeping similar computational costs. This analysis indicates that the proposed methods may have better accuracy and stability properties, such as, for example, larger stability regions in the case of explicit methods, or stage order greater than one for singly diagonally implicit methods.
The possibility of identifying good families of methods with a larger number of degrees of freedom can also have implications in the field of time discretization of partial differential equations. For example, Self Starting GLMs allow the determination of new efficient and highly stable Implicit-Explicit and Strong Stability Preserving methods.
Finally, we report numerical experiments which confirm that Self Starting GLMs are competitive with Runge-Kutta methods and can have better performance on nonstiff, mildly stiff and stiff problems.