A new framework for polynomial approximation to differential equations

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We discuss a new framework for the polynomial approximation to the solution of initial value problems for ordinary differential equations (ODEs)

$$\dot{y}(t) = f(t, y(t)), \qquad t \in [t_0, T], \qquad y(t_0) = y_0 \in \mathbb{R}^m, \tag{1}$$

and delay differential equations (DDEs) in the form,

$$\begin{array}{rcll} \dot{y}(t) & = & f(t, y(t), y(t-\tau)), & t \in [t_0, T], \\ y(t) & = & \phi(t), & t \in [t_0 - \tau, t_0], \end{array} \tag{2}$$

The framework is based on a truncated expansion of the vector field along an orthonormal basis, which projects the differential problem onto a finite dimensional vector space. This procedure leads to a new class of numerical methods that may be regarded as a perturbation of the original differential problem. Consequently, a perturbation analysis has been successfully employed to understand how the solutions of the two problems are related.
The approach has been initially devised for problem (1) ([1, 2, 3]), and very recently extended to delay differential equations in the form (2) ([5, 4]). Relevant classes of Runge-Kutta methods can be derived within this framework.

References

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