Perturbative analysis of numerical discretization to stochastic Hamiltonian problems

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This talk will highlight recent results based on the study of numerical dynamics associated to discretization of stochastic Hamiltonian problems, since they are excellent models useful in a large number of applications, when the dynamics is subject to random perturbations. In particular, stochastic Hamiltonian problems are the most suitable candidates to conciliate classical Hamiltonian mechanics with the non-differentiable Wiener process, which describes the continuous innovative character of stochastic diffusion.
Our analysis is focused on the study of stochastic Runge-Kutta methods developed by Burrage and Burrage, obtained as a stochastic perturbation of classical symplectic Runge-Kutta methods. In particular, we are interested in understanding whether these methods are capable to maintaining the linear drift visible in the expected value of the Hamiltonian. The analysis shows Runge-Kutta methods present an error that increases with the parameter ϵ, being ϵ the amplitude of the diffusive part of the problem. Through a perturbative theory, we investigate the reason of this behaviour, due to the presence of a secular term $\epsilon\sqrt{t}$ that destroying the overall conservation accuracy. Numerical tests confirm the theoretical analysis.

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