Numerical conservations features of stochastic Hamiltonian systems

Raffaele D'Ambrosio, **Stefano Di Giovacchino**

Department of Information Engineering and Computer Science and Mathematics
University of L'Aquila (Italy)
raffaele.dambrosio@univaq.it, stefano.digiovacchino@graduate.univaq.it

We address our attention to the numerical approximation of stochastic Hamiltonian systems, both of Itô and Stratonovich types. It has been shown that, in the Itô case, a trace equation is satisfied, describing the linear growth of the expected Hamiltonian in time [1], while energy conservation is visible in the Stratonovich setting [5]. The first part of this talk is devoted to the analysis of the effectiveness of Monte Carlo estimates employed in the application of drift-preserving numerical schemes for Itô Hamiltonian systems [3], i.e., methods able to reproduce the trace equation along the numerical dynamics [1, 2]. In the second part of this talk, we aim to provide a characterization of the long-term behavior of numerical discretizations to such stochastic Hamiltonian systems [4] by means of the so-called *weak backward error analysis*. The key ingredient is the construction of weak stochastic modified equations associated to such numerical methods [4, 6, 7]. Finally, numerical experiments are also provided to confirm the theoretical results.

References

[1] P.M. Burrage, K. Burrage, *Structure-preserving Runge-Kutta methods for stochastic Hamiltonian equations with additive noise*, Numer. Algorithms 65(3), 519–532 (2014).

[2] C. Chen, D. Cohen, R. D'Ambrosio, A. Lang, *Drift-preserving numerical integrators for stochastic Hamiltonian systems*, Adv. Comput. Math. 46(2), 27 (2020).

[3] R. D'Ambrosio, S. Di Giovacchino, *How do Monte Carlo estimates affect stochastic geometric numerical integration?*, submitted (2021).

[4] R. D'Ambrosio, S. Di Giovacchino, *Long-term analysis of stochastic Hamiltonian systems under time discretizations*, submitted (2021).

[5] G. N. Milstein, YU. M. Repin, M.V. Tretyakov, *Numerical Methods for Stochastic Systems Preserving Symplectic Structure*, SIAM J. Numer. Anal. 40, 1583–1604 (2002).

[6] T. Shardlow, *Modified equations for stochastic differential equations*, BIT Numer. Math. 46, 111–125 (2006).

[7] K.C. Zygalakis, *On the existence and the applications of modified equations for stochastic differential equations*, SIAM J. Sci. Comput. 33, 102–130 (2011).