Search for implicit-explicit general linear methods with inherent Runge-Kutta stability

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Many practical problems in science and engineering are modeled by large systems of ordinary differential equations (ODEs) which arise from discretization in space of partial differential equations (PDEs) by finite difference methods, finite elements or finite volume methods, or pseudospectral methods. For such systems there are often natural splittings of the right hand sides of the differential systems into two parts, one of which is non-stiff or mildly stiff, and suitable for explicit time integration, and the other part is stiff, and suitable for implicit time integration. The efficient solution can be provided by implicit-explicit (IMEX) schemes.
In present research we consider the class of general linear methods (GLMs) for ordinary differential equations. We construct IMEX GLMs of order $p = 1, 2, \ldots, 4$ with desired stability properties. We assume that the explicit and implicit parts of the IMEX scheme have the same abscissa vector $\mathbf{c}$ and coefficient matrices $\mathbf{U}$ and $\mathbf{B}$. We look for implicit methods of order p and stage order $q = p$, which have the property of so-called inherent Runge–Kutta stability (IRKS), and which are A-stable and if possible, also L-stable. We also require that the vector of external approximation is of Nordsieck form. Next, we attempt to maximize the combined region of absolute stability. Finally, we apply constructed methods to a series of test problems.

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References

[1] U.M. Ascher, S.J. Ruuth, R.J. Spiteri, *Implicit-explicit Runge–Kutta methods for time-dependent partial differential equations*, Appl. Numer. Math., 25 (1997), 151—167.

[2] M. Braś, A. Cardone, Z. Jackiewicz, P. Pierzchała, *Error propagation for implicit-explicit general linear methods*, Appl. Numer. Math., 131 (2018), pp. 207–231.

[3] A. Cardone, Z. Jackiewicz, A. Sandu, H. Zhang, *Extrapolated implicit–explicit Runge–Kutta methods*, Math. Model. Anal., 19 (2014), pp. 18—43.

[4] A. Cardone, Z. Jackiewicz, A. Sandu, H. Zhang, *Extrapolation-based implicit–explicit general linear methods*, Numer. Algorithms, 65 (2014), pp. 377—399.