Asymptotic preserving and asymptotic accurate schemes for hyperbolic systems with stiff hyperbolic or parabolic relaxation

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Several systems of evolutionary partial differential equations may contain stiff terms, which require an implicit treatment. Typical examples are hyperbolic systems with stiff hyperbolic or parabolic relaxation characterized by a relaxation parameter ε. In the hyperbolic-to-hyperbolic relaxation (HSHR) a natural treatment consists in adopting implicit-explicit (IMEX) schemes, in which the relaxation is treated by an implicit scheme, while the hyperbolic part is treated explicitly [1]. In the hyperbolic-to-parabolic relaxation (HSPR) standard IMEX methods relax to an explicit scheme for the parabolic limit, thus suffering from parabolic CFL restriction. In [2, 3] this drawback has been overcame by a penalization method, consisting in adding and subtracting the same term, so that the system appears as the limit relaxed system plus a small perturbation. Furthermore, in [4] a unified IMEX approach has been introduced for systems which may admit both limits. This latter approach generalizes the two ones: HSHR and HSPR. All these approaches are capable to capture the correct asymptotic limit of the system when $\varepsilon \to 0$, i.e., the scheme is *asymptotic preserving* (AP) independently of the scaling used. However, the AP property guarantees only the *consistency* of the scheme in the stiff limit $\varepsilon \to 0$, but it does not not imply in general that the scheme preserves the order of accuracy in time in the limit and the order of accuracy may drop to low orders. In the literature of hyperbolic system with stiff relaxation this *order reduction phenomenon* is extensively studied and cured, see for example [1]. A scheme that preserves the order of accuracy in time in the limit is said asymptotically accurate (AA). In this talk we show that under several assumptions on the IMEX scheme, all the numerical approaches used to solve hyperbolic systems with stiff relaxation are both AP and AA, i.e., they maintain the correct order of accuracy of the original IMEX scheme in the limit of the relaxation parameter ε.

References

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