Adaptive Exponential Splitting

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For the efficient numerical integration of evolutionary differential equations with additive right-hand side,

$$\partial_t u = F_1(t,u) + F_2(t,u) + \dots,$$

approximation by exponential splitting methods based on the solution and systematic recombination of sub-problems is often a convenient choice. For the purpose of reliable adaptive stepsize control, several techniques for practical local error estimation are introduced and discussed in [1], and further works are devoted to particular applications, e.g., Schrödinger type problems or hyperbolic problems [2]. The employed techniques comprise optimized pairs of schemes, Milne-type estimators, and defect-based approaches, and are used with schemes of various approximation orders. Also, more than two suboperators are treated, see [2, 3] for the case of three sub-operators F_j. Coefficients of various optimized methods are collected at [4].

In the simulation of magnetohydrodynamics, it is natural to split the vector field into 4–8 operators, even. Additionally, certain positivity conditions on the coefficients need to be satisfied. We discuss the construction of methods and present numerical results for hyperbolic test problems.

References

[1] W. Auzinger, H. Hofstätter, D. Ketcheson, O. Koch, *Practical splitting methods for the adaptive integration of nonlinear evolution equations. Part I: Construction of optimized schemes and pairs of schemes*, BIT Numer. Math. 57/1 (2017), pp. 55–74.

[2] W. Auzinger, O. Koch, M. Thalhammer, *Defect-based local error estimators for splitting methods, with application to Schrödinger equations, Part II. Higher-order methods for linear problems*, J. Comput. Appl. Math. 255 (2014), pp. 384–403.

[3] W. Auzinger, O. Koch, M. Thalhammer, *Defect-based local error estimators for high-order splitting methods involving three linear operators*, Numer. Algor. 70/1 (2015), pp. 61–91.

[4] https://www.asc.tuwien.ac.at/auzinger/splitting/