Convergence of the piecewise orthogonal collocation for computing periodic solutions of delay equations

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Realistic models from several areas of biology, such as structured population dynamics or epidemiology, are often based on delay equations. Due to the infinite dimension of the relevant dynamical systems, the dynamics of such models can generally not be studied analytically and must then be approximated numerically. An important target in this context is represented by the computation of periodic solutions. As for Retarded Functional Differential Equations (RFDEs), the classical piecewise orthogonal collocation proposed in [4] for computing periodic solutions has only recently been proven to be convergent in [2], following the abstract approach in [5] for general boundary value problems defined by delay equations. The method can also be extended to Renewal Equations (REs), and its convergence is proved in [1, 3]. We take a further step by describing the extension of the method to coupled systems of RFDEs and REs. In this talk, I present the main ideas behind the proof of its convergence and emphasize the challenges that emerge with respect to the cases of RFDEs and REs separately. Finally, I show some numerical experiments that further validate the theoretical results.

References

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