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Prony-Type Polynomials

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Let us consider N-sparse bivariate exponential sum,

$$f(\mathbf{k}) = \sum_{j=1}^{N} a_j \exp\left(-\mathrm{i}\langle \boldsymbol{\omega}_j, \mathbf{k}\rangle\right) + \epsilon(\mathbf{k}),$$

where $a_1, \ldots, a_N \in \mathbb{C}\backslash\{0\}$, $\mathbf{k} \in \mathbb{Z}_+^2$, $\langle \boldsymbol{\omega}_j, \mathbf{k}\rangle$ denotes the inner product of $\mathbf{k}$ and $\boldsymbol{\omega}_j$, and $\epsilon(\mathbf{k})$ is are a random variable. The problem of parameter estimation of the exponential sum is to determine approximately elements $\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_N \in (0, 2\pi]^2$ out of finitely many noisy samples of f.

Inspired by the one-dimensional approach developed in [1], we propose to use the method of Prony-type polynomials, when the parameters $\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_N$ can be recovered as a set of common zeros of Prony-type polynomials, some bivariate polynomials of an appropriate multi-degree. Numerical experiments show the PTP method is more stable in the presence of noise than other methods. Moreover, using an autocorrelation sequence in the PTP approach allows us to improve significantly the stability of the method in the noisy data case.

This is joint work with Jürgen Prestin (Institute of Mathematics, University of Lübeck).

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[2] S. Kunis, T. Peter, T. Römer, U. von der Ohe. A Multivariate Generalization of Prony's Method. *Linear Algebra Appl.* **490**, (2016), p. 31-47.

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