Direct method for function approximation on data defined manifolds, II

Hrushikesh Mhaskar[a], **Ryan O'Dowd**[a]

[a]Institute of Mathematical Sciences, Claremont Graduate University (United States)

hrushikesh.mhaskar@cgu.edu, ryan.o'dowd@cgu.edu

In theoretical analysis of function approximation in the context of machine learning, one of the standard assumptions in order to avoid the curse of dimensionality is the manifold assumption; i.e., one assumes that the data is sampled from an unknown sub-manifold of a high dimensional Euclidean space. A great deal of research deals with obtaining information about this manifold, such as the eigen-decomposition of the Laplace-Beltrami operator or coordinate charts. The theory of function approximation based on this preliminary information is also well studied. Since the manifold is unknown, this two step approach implies some extra errors in the approximation stemming from the approximation of the basic quantities from the data in addition to the errors inherent in function approximation. In [1], HNM has proposed a one-shot direct method to achieve function approximation without knowing anything about the manifold other than its dimension. However, one cannot pin down the class of approximants used in that paper.

In this paper, we view the unknown manifold as a sub-manifold of an ambient hypersphere and study the question of constructing a one-shot approximation using the spherical polynomials based on the hypersphere; again, our approach does not require pre-processing of the data to obtain information about the manifold other than its dimension. We give optimal rates of approximation for relatively "rough" functions. The class of approximants is the restriction of the spherical polynomials to the unknown manifold, but we need not use this fact in our construction.

References

[1] H. N. Mhaskar. A direct method for function approximation on data defined manifolds. *Neural Networks*, 132:253–268, 2020. ArXiv preprint arXiv:1908.00156, doi:10.1016/j.neunet.2020.08.018.