An algorithm for constrained regression by penalized splines

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Regression splines are confirmed powerful and versatile to investigate the data structures, and predict data behaviour. Different penalized models are available in literature, so-called since balancing the regression fidelity and a weighted penalizing term. The most commonly used are P-splines [5] that are based on two main ingredients: polynomial B-splines and a second order discrete difference penalty. More recently [2] a new class of penalized splines, said hyperbolic-polynomial splines (HP-splines), combines the advantages of P-splines with the idea of a data-driven selection of space parameters, making them more suitable for capturing exponential trends. Actually, P-splines can be recognized as one of the most successfully smoothers used in a wide range of applications. For example, in social and behavioural sciences, constrained P-splines are definite based on theoretical hypotheses regarding their shape and monotonicity, translated into local and global constraints on the successive derivatives of the functional model [1]. Moreover, P-splines are used for Bayesian spectral density estimation; e.g. in [6] the authors propose statistic techniques for a data-driven selection of the spline knots placement, so giving up the uniform distribution of the knots which guarantees analytical reproduction properties (assured both for P-splines [4] and for HP splines [3]). We formulate a constraint optimization problem to make a penalized spline positive, so suitable for density estimation, and propose a greedy-type algorithm to dynamically tune the model shape, while preserving positiveness and space reproduction properties. A theoretical result concerning the bounded variation of the P-spline model is also given and drives the selection.

References

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