L_p Markov exponent of certain UPC sets

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We say that a compact set $\emptyset \neq E \subset \mathbb{R}^m$ satisfies L_p Markov type inequality (or: is a L_p Markov set) if there exist $\kappa, C > 0$ such that, for each polynomial $P \in \mathcal{P}(\mathbb{R}^m)$ and each $\alpha \in \mathbb{N}_0^m$,

$$\|D^\alpha P\|_{L_p(E)} \leq (C(\deg P)^\kappa)^{|\alpha|} \|P\|_{L_p(E)}, \tag{1}$$

where $D^\alpha P = \frac{\partial^{|\alpha|} P}{\partial x_1^{\alpha_1} \ldots \partial x_m^{\alpha_m}}$ and $|\alpha| = \alpha_1 + \cdots + \alpha_m$.

Clearly, by iteration, it is enough to consider in the above definition multi-indices α with $|\alpha| = 1$. The inequality (1) is a generalization of the classical Markov inequality:

$$\|P'\|_{C([-1,1])} \leq (\deg P)^2 \|P\|_{C([-1,1])}.$$

In this talk we shall consider the following problem:

For a given L_p Markov set E determine $\mu_p(E) := \inf\{\kappa : E$ satisfies (1)$\}$.

Our goal is to establish L_p Markov exponent of the following domains

$$K := \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1,\ ax^k \leq y \leq f(x)\},$$

where $k \in \mathbb{N}$, $k \geq 2$, $a > 0$ and $f : [0, 1] \to [0, \infty)$ is a convex function such that $f(1) > a$, $f'(0) = f(0) = 0$, $f'(1) < \infty$, and $(f)^{1/k}$ is a concave function on the interval $(0, 1)$.

References

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