On the best constant in Ulam stability

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A linear operator is Ulam stable if for every approximate solution of the associated equation there exists an element in the kernel of the operator close to the approximate solution. The linear differential operator with constant coefficients

$$D(y) = y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y, \quad y \in \mathcal{C}^n(\mathbb{R}, X)$$

acting in a Banach space X is Ulam stable if and only if its characteristic equation has no roots on the imaginary axis. We prove that if the characteristic equation of D has distinct roots r_k satisfying $\Re r_k > 0$, $1 \leq k \leq n$, then the best Ulam constant of D is

$$K_D = \frac{1}{|V|} \int_0^\infty \Big| \sum_{k=1}^{n} (-1)^k V_k e^{-r_k x} \Big| dx,$$

where $V = V(r_1, r_2, \ldots, r_n)$ and $V_k = V(r_1, \ldots, r_{k-1}, r_{k+1}, \ldots, r_n)$, $1 \leq k \leq n$, are Vandermonde determinants.