MODULUS OF CONTINUITY AND A JACKSON-TYPE INEQUALITY ON MANIFOLDS

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ABSTRACT. We consider a Hilbert space $\mathbf{H}$ equipped with a set of strongly continuous bounded semigroups satisfying certain conditions. The conditions allow to define a family of moduli of continuity $\Omega^r(s, f)$, $r \in \mathbb{N}, s > 0$, of vectors in $\mathbf{H}$ and a family of Paley-Wiener subspaces PW_σ parametrized by bandwidth $\sigma > 0$. These subspaces are explored to introduce notion of the best approximation $\mathcal{E}(\sigma, f)$ of a general vector in $\mathbf{H}$ by Paley-Wiener vectors of a certain bandwidth $\sigma > 0$. The main objective of the paper is to prove the so-called Jackson-type estimate $\mathcal{E}(\sigma, f) \leq C\left(\Omega^r(\sigma^{-1}, f) + \sigma^{-r}\|f\|\right)$ for $\sigma > 1$. Our assumptions are satisfied for a strongly continuous unitary representation of a Lie group G in a Hilbert space $\mathbf{H}$. It allows to obtain the Jackson-type estimates on homogeneous manifolds.

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