Regularity and Stability for a Convex Feasibility Problem

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The 2-sets convex feasibility problem aims at finding a point in the nonempty intersection of two closed convex sets A and B in a Hilbert space H. The method of alternating projections is the simplest iterative procedure for finding a solution and it goes back to von Neumann. Originally, he proved that the method of alternating projection converges when A and B are closed subspace. Then, for two generic convex sets, the weak convergence of the alternating projection was proved by Bregman in 1965 ([2]). Nevertheless, the problem of whether the alternating projections algorithm converges in norm for each couple of convex sets remained open till the counterexample given by Hundal in 2004. This example shows that the alternating projections do not always converge in norm. One of the most important sufficient condition ensuring the norm convergence of the alternating projections algorithm is based on the so called "regularity" property for the couple (A, B) ([1]). In this talk we consider two sequences of closed convex sets $\{A_n\}$ and $\{B_n\}$, each of them converging, with respect to the Attouch-Wets variational convergence, respectively, to A and B. Given a starting point a_0, we consider the sequences of points obtained by projecting on the "perturbed" sets, i.e., the sequences $\{a_n\}$ and $\{b_n\}$ defined inductively by $b_n = P_{B_n}(a_{n-1})$ and $a_n = P_{A_n}(b_n)$. i.e., for each $\{a_n\}$ and $\{b_n\}$ as above we have $\operatorname{dist}(a_n, A \cap B) \to 0$ and $\operatorname{dist}(b_n, A \cap B) \to 0$. We show that the regularity of the couple (A, B) implies not only the norm convergence of the alternating projections sequences for the couple (A, B), but also that the couple (A, B) is d-stable.i.e., for each $\{a_n\}$ and $\{b_n\}$ as above we have $\operatorname{dist}(a_n, A \cap B) \to 0$ and $\operatorname{dist}(b_n, A \cap B) \to 0$. Similar results are obtained also in the case $A \cap B = \emptyset$, considering the set of best approximation pairs instead of $A \cap B$. Finally, under appropriate geometrical and topological assumptions on the intersection of the limit sets, we ensure that the sequences $\{a_n\}$ and $\{b_n\}$ converge in norm to a point in the intersection of A and B. The talk is based on [4] and [3].

References

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