Variational problems with nonconstant gradient constraints

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Aim of the talk is to study a nonlinear variational problem with gradient constraints and homogeneous Dirichlet boundary condition.

Variational problems with gradient constraints have been intensively studied a few decades ago and have seen many progresses also recently (see [2] for an overview on free boudary problems and applications related to the non-constant gradient constrained problem). Indeed, an important example among them is the well-kwon elastic-plastic torsion problem. An interesting property of gradient constrained variational problems is the connection with double obstacle problems, even if this equivalence is not true in the general case (see [1], and counterexamples in [3, 4] in the case of nonconstant gradient constraint $|Du| \leq g(x)$). In the talk we show that a nonlinear monotone variational inequality with convex gradient constraints is equivalent to a double obstacle problem.

The existence of Lagrange multipliers is also proved.

The proof is based on a new strong duality principle, that works in infinite dimensional settings.

References

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