

# Completeness theorems for systems of particular solutions of partial differential equations

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The results presented here fit in the problem of the completeness as proposed by Picone. This means to show that a certain system of particular solutions of a PDE is complete in a functional space on the boundary of a domain. This approach to the problem of polynomial approximation is more sophisticated than the usual one, which extends the classical Mergelyan Theorem. These results are related not only to a partial differential equation, but also to a particular boundary value problem. In particular we present necessary and sufficient conditions for the completeness of polynomial solutions of a partial differential equations of higher order in any number of variables related to the Dirichlet problem. We shall give also some recent results obtained for systems, where very little is known.

## References

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