On Arzelà-Ascoli Theorem. Dualizing compactness property.

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We consider a metric space (U, ρ), two abitrary nonempty sets X, Y, a map (duality map)

$$D : X \times Y \to U$$

such that the partial functions D_x, D_y on Y, respectively X are totally bounded.
If d', d'' are the metrics on X, respectively Y given by:

$$d'(x', x'') = \sup\{\rho\left(D_{x'}(y), D_{x''}(y)\right); y \in Y\}$$

$$d''(y', y'') = \sup\{\rho\left(D_{y'}(x), D_{y''}(x)\right); x \in X\}$$

then the following assertions hold:

1. *The metric space* (X, d') *is totally bounded iff the metric space* (Y, d'') *is totally bounded.*

2. *The family* $(D_x)_x$ *of functions on* Y *has "finite small oscillations" iff the family* $(D_y)_y$ *has a similar property on* X.

The well-known assertions: Schauder theorem on compact linear operators, Scorohad compactness criterion with respect to a specific distance on trajectories, the famous Arzelà -Ascoli theorem...may be derived from the above assertions.

References

[1] N. Bourbaki, *Espaces vectoriels topologiques*, Hermann, Paris, 1966.

[2] G. Choquet, *Lectures on analysis*, London - Amsterdam - Benjamin, 1969.

[3] N. Bourbaki, A. Cornea, *Potential Theory on Harmonic Spaces.*, Springer, 1972.

[4] J. L. Doob, *Potential Theory and Probabilistic counterpart*, Berlin - Heidelberg - New York: Springer, 1984.

[5] B. E. Dynkin, *Markov processes*, Berlin - Heidelberg - New York: Springer, 1965