Evolution equations with superlinear growth

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In this talk we present existence results [1] for mild solutions of the following partial differential equation of parabolic type

$$u_t = \Delta u + h(t, x, u(t, x)) \quad \text{for} \quad (t, x) \in (0, T) \times \Omega$$

coupled with Dirichlet boundary conditions on $\partial\Omega$ and a nonlocal initial condition $u(0, \cdot) = g(u)$ described by a map $g : C([0, T]; L^p(\Omega)) \to L^p(\Omega)$ with $2 \leq p < \infty$, where $\Omega \subset \mathbb{R}^k$ is a bounded domain with C^2-boundary. Here $h\colon [0, T] \times \Omega \times \mathbb{R} \to \mathbb{R}$ is a given map with superlinear growth. The nonlocal condition includes as particular cases the Cauchy multipoint problem, the weighted mean value problem, and the periodic problem. Existence results are obtained by means of a Leray-Schauder continuation principle, transforming the above problem to an ordinary differential equation in the abstract setting given by the Banach space $L^p(\Omega)$. Handling superlinear growth in this context is particularly challenging since the Nemytskii operator associated to the Carathéodory function $h : \Omega \times \mathbb{R} \to \mathbb{R}$ maps the space $L^p(\Omega)$ continuously on itself if and only if h is sublinear, as stated in Vainberg's theorem. We overcome this difficulty exploiting the compactness and the regularity properties of the semigroup generated by the Laplacian operator and constructing a suitable approximation technique. This approach was developed in [2] for the Cauchy problem and is extended here to the nonlocal one.

References

[1] I. Benedetti, S. Ciani, *Evolution equations with nonlocal initial conditions and superlinear growth*, Journal of Differential Equations, 318 (2022), pp. 270–297.

[2] I. Benedetti, E.M. Rocha, *Existence results for evolution equations with superlinear growth*, Topological Methods in Nonlinear Analysis, 54 no. 2B (2019), pp. 917–36.