How to measure the noncompactness of operators

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There are various methods to measure how "far" an operator in a normed space is from being compact. The most important tools are the essential norm (for linear operators) and the measure of noncompactness (for nonlinear operators).

We illustrate this by means of two linear operators, viz. the *multiplication operator*

$$M_\mu x(t) := \mu(t)x(t) \qquad (\mu : [0,1] \to \mathbb{R} \text{ given})$$

and the *substitution operator*

$$S_\varphi x(t) := x(\varphi(t)) \qquad (\varphi : [0,1] \to [0,1] \text{ given})$$

in the function spaces $C[0,1]$ with norm $\|x\|_C := \max\{|x(t)| : 0 \leq t \leq 1\}$ and $BV[0,1]$ with norm $\|x\|_{BV} := |x(0)| + var(x; [0,1])$. Combing this with analogous results for nonlinear superposition operators one may obtain existence results for boundary value problems.

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