A fourth order scheme for nonlocal diffusive equations

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For some $a > 0$, in $x \in [-a, a]$, $t \geq 0$, we consider the diffusion equation with a nonlocal type kernel, where the function $\varphi(\cdot)$ is assumed to be sufficiently smooth:

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2} - u(x,y)J(u,x,t), \quad J(u,x,t) = \int_{-\infty}^{\infty} \varphi(y-x)u(y,t)\, dy \tag{1}$$

and with suitable initial and boundary conditions.
We propose a numerical method which is a combination of a "discrete-collocation" method in space and of the classical Runge-Kutta 4-5 method in time. We collocate the equation on a set P of selected equispaced points in $[-a, a]$. The second partial derivative with respect to space is discretized by suitable divided difference schemes of order 4. The integral J is approximated by means of a quadrature formula based on the generalized (iterated) Bernstein polynomial approximation, with the advantage of using the same set P of equispaced points in $[-a, a]$. The main feature of this formula lies in the high approximation order for smooth functions: more precisely, if the function to be integrated has $2r$ continuous derivatives, the convergence order of the quadrature rule is r. Matching this feature with the order chosen for the discretization of the derivatives, we obtain a system of ordinary differential equations in time. We finally proceed to integrate it by applying the Runge-Kutta scheme of order 4-5. Some numerical results will be shown.