Software Implementation of the Partition of Unity Method

Choosing the Shape Parameter Optimally in the RBF Collocation Method

Lin-Tian Luh[a]

[a] Department of Data Science, Providence University, Taiwan
ltluh@pu.edu.tw

In this talk we totally discard the traditional trial-and-error algorithms of choosing acceptable shape parameter c in the multiquadrics

$$\phi(x) := -\sqrt{c^2 + \|x\|^2}$$

when dealing with differential equations. Instead, we predict the optimal value of c directly by the MN-curve theory and hence avoid the time-consuming steps of solving a linear system required by each trial of the c value in the traditional methods. The quality of the c value thus obtained is supported by the newly born rigorous choice theory of the shape parameter. Experiments show that the approximation error of the approximate solution to the differential equation is very close to the best approximation error among all possible choices of c.

The differential equation dealt with is the Poisson equation. A standard 3D Poisson equation is of the form

$$\begin{cases} u_{xx}(x,y,z) + u_{yy}(x,y,z) + u_{zz}(x,y,z) = f(x,y,z) & \text{for } (x,y,z) \in \Omega \backslash \partial\Omega, \\ u(x,y,z) = g(x,y,z) & \text{for } (x,y,z) \in \partial\Omega, \end{cases}$$

where Ω is the domain with boundary $\partial\Omega$, and $f,\ g$ are given functions. A natural extension to d dimensions can be easily understood by replacing (x,y,z) with $(x_1,\cdots,x_d)$ and letting $\Omega \subseteq R^d$. The approximate solution will be of the form

$$\hat{u}(x) := \sum_{i=1}^{N} \lambda_i \phi(x - x_i) + p(x),\ x \in R^d,$$

where $\phi(x)$ is just the multiquadrics, $p(x)$ is a zero degree polynomial in R^d and hence a constant λ_0, and $X = \{x_1, \ldots, x_N\}$ is a set of points scattered in the domain without meshes. An unorthodox way even drops λ_0 and lets it be 0.The constants $\lambda_i,\ i = 0, \cdots, N$, are chosen so that $\hat{u}(x)$ satisfies the differential equation (including boundary conditions) at the points $x_i,\ i = 1, \cdots, N$, called the collocation points. The parameter c is chosen according to the MN curves which can be easily sketched by Matlab or Mathematica. If the minimum of the function MN(c) occurs at $c = c^*$, then c^* is just the optimal choice of c.