Moving Least Square Approximation using Variably Scaled Discontinuous Weight Functions

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The moving least square (MLS) is an approximation (of low order) method introduced by Shepard [4] and generalized to higher approximation order by Bos and Salkauskas [1]. The MLS method approximates a function given at irregularly spaced points by weighted least square approximations. The smoothness of the MLS approximant is decided by the smoothness of the weight functions (cf. [5]). Moreover, the weight function are considered to be smooth functions of some order, regardless of the smoothness of the underlying function to be approximated. However, in case that the underlying functions possess some discontinuities at some points, smooth approximants become highly oscillatory near the discontinuities.
In this talk we show how to choose the weight function(s) so that the approximant reflects the discontinuities in the data. For doing so, we consider piecewise weight functions,of some order $\ell + 1$ of smoothness, that are themselves discontinuous. We take the weight functions as *Variable Scaled Discontinuous Kernels*, recently introduced in [2, 3], that enable us to reconstruct jump discontinuities. We will see that, this choice of weight functions provide tailored approximant that is useful to avoid overshoots near the edges of the underlying functions. Both theoretical and numerical analysis is provided.

References

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