Exploring the expressive power of ExSpliNet, a new spline-based neural network model

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The approximation of high-dimensional functions is an extremely challenging task. Classical methods, based on a mesh, suffer in general from the so-called *curse of dimensionality* and thus in practice they are only suited for addressing lower-dimensional problems. On the other hand, machine learning techniques that are mesh-free, and in particular *(deep) neural networks*, seem to be able to overcome this issue for different kinds of high-dimensional problems, especially in the context of image analysis and pattern recognition.

The *ExSpliNet model*, introduced in [1], is an interpretable and expressive neural network model that combines ideas of Kolmogorov neural networks, ensembles of probabilistic trees, and multivariate B-spline representations. The model, inspired by Kolmogorov's Superposition Theorem, uses univariate splines as inner functions that feed L-variate tensor-product splines as outer functions, all of them represented in terms of B-splines. Here, L is supposed to be not too high. The inner functions act as low-dimensional feature extractors, while the outer functions can be regarded as probabilistic trees. The ExSpliNet model can be efficiently evaluated thanks to the computational properties of B-splines. In [1], the effectiveness of the proposed model was also tested on classical machine learning benchmark datasets and it was numerically illustrated that the model is particularly suited for data-driven (smooth) function approximation and to face differential problems, in the spirit of physics-informed neural networks.

In this talk, after discussing the ExSpliNet model's definition and its main properties, we focus on the approximation capabilities and present two constructive results that mitigate the curse of dimensionality. More precisely, following ideas similar to the ones proposed in [2, 3], we obtain error bounds for the ExSpliNet approximation of a subset of multivariate continuous functions and also of multivariate generalized bandlimited functions. The curse of dimensionality is lessened in the first case, while it is completely overcome in the second case.

References

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