Optimal spline spaces are outlier free

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Smooth isogeometric methods obtain an extremely good spectral approximation of differential operators when compared to classical finite element methods, however, they still present a few poorly approximated eigenvalues and eigenfunctions which are referred to as outliers. This superior spectral behavior translate into improved numerical simulations, especially for explicit dynamics, but the presence of outliers dampens the possible gain. In this talk we explain how isogeometric discretizations using the optimal spline spaces identified in [1] lead to outlier free approximations of eigenvalue problems. This talk is based on the results of [2, 3].

References

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Collocation Isogeometric Approximation of acoustic wave problems

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In this presentation we consider the numerical approximation of acoustic wave problems with absorbing boundary conditions by the Isogeometric discretization in space, and Newmark scheme in time, both explicit and implicit [2, 5]. Isogeometric Analysis (IGA) allows not only the standard p- and hp- refinement of hp- finite elements and spectral elements, where p is the polynomial degree of the C^0 piecewise polynomial basis functions, but also a novel k- refinement where the global regularity k of the IGA basis functions is increased proportionally to the degree p, up to the maximal IGA regularity $k = p - 1$ [2].

In the framework of NURBS-based IGA, first we have considered Galerkin approaches [4] and then we have moved on to collocation methods, that in general optimize the computational cost, still taking advantage of IGA geometrical flexibility and accuracy [1, 3].
Proper boundary conditions are also considered. While homogeneous Neumann conditions provide a good mathematical representation of a free surface, absorbing boundary conditions are imposed in order to simulate wave propagation in infinite domains, by truncating the original unbounded region into a finite one.
Several numerical examples illustrate the stability and convergence properties of the numerical collocation IGA methods with respect to all the IGA approximation parameters, namely the local polynomial degree p, regularity k, mesh size h, and to the time step size Δt of the Newmark schemes [5]. Some numerical results on the spectral properties of the Collocation IGA mass and stiffness matrices are also presented.

References

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