Efficient Quadrature in Isogeometric Galerkin methods and Isogeometric Boundary Element methods

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We present the construction of quadrature rules for the approximation of integrals that occur when elliptic problems are numerically solved using Isogeometric Analysis (IgA) both in the standard Galerkin and in the Galerkin Boundary Element frameworks.
In particular the IgA Galerkin discretization askes for the computation of integrals involving splines, and a new formation and assembly strategy was proposed which resulted in significant speedups in the formation and assembly time of the Galerkin mass matrix, see [3]. Moreover, recurrence relations can be used in order to require exactness of the rules also in cases where singularity occurs, as in the case of BEMs [1, 2].
Moreover, we discuss various important details for the practical implementation of the quadrature formation strategies proposed in [1, 2, 3, 4]. Specifically, we discuss the weighted quadrature scheme to accurately integrate the elements of the stiffness matrix and we discuss efficient assembly in the BEM case where singular integrals appear. We will review such approaches and then focus on the use of spline quasi-interpolation to approximate integrand factors, that gives a final formulation where integrals can be evaluated evaluated via recurrence relations, as proposed recently in [4]. Considered cases include hypersingular and singular integrands. Convergence results of the proposed quadrature rules are given, with respect to both smooth and non smooth integrands. Numerical tests confirm the behavior predicted by the analysis.
We show that the accuracy is maintained while the computational burden of forming the matrix equations is significantly reduced.
The research is a part of collaborations with A. Aimi, A. Falini, M.L. Sampoli, A. Sestini, G. Sangalli, M. Tani.

References

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